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The shape of a diffracting shock wave

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This paper describes an experimental study of the shape of a shock diffracting around a corner made up of two plane walls, for corner angles from 15 to 165° (in 15° steps) and shock Mach numbers from $M_0 = 1{\cdot}0$ to $4{\cdot}0$. The results are compared with profiles determined from the diffraction theory of Whitham (1957, 1959). The agreement is shown to be good for an incident shock Mach number of 3·0, and fair in other cases. The behaviour is found to follow the trends established by Lighthill (1949) in a linearized theory. Results for the Mach number of the wall shock are also presented. The shock does not degenerate to a sound wave even for large corner angles and low Mach numbers.

1. Introduction

The only theory, of which the author is aware, which predicts the shape of a shock wave, of any strength, diffracting around a corner of any angle, is the approximate theory of Whitham. In this theory the shock positions are denoted by curves of constant $\alpha = a_0 t$, where a_0 is the sound speed in the undisturbed region ahead of the shock and t is the time elapsed from when the shock reached the corner. For a plane walled convex corner the solution comprises a centred characteristic fan, over which the shock is curved, separating two regions in which the shock is plane, as shown in figure 1. The curved part of the shock is independent of the corner angle, and the wall shock (the plane portion after the corner) is tangent to this curve and perpendicular to the wall. Detailed solutions to this theory for a number of wall shapes and incident shock Mach numbers between 1·0 and 5·0 have been given (Skews 1966). The present paper compares these solutions with experimentally determined shock wave profiles.

The experiments were conducted in an air/air shock tube having a contraction in the channel in order to increase the shock Mach number. The final 3 in. wide by 2 in. high channel section opens out into a 10 in. diameter working section in which the models were mounted. A 5-channel light source with variable time delays, in a 10 in. field schlieren system, was used to record the phenomenon. Two typical photographic records are presented in figure 2, plate 1.

A discussion of the perturbed region behind the shock will be given in a later paper.

2. Shock wave profiles

The first consideration was to determine whether, for a given corner and initial Mach number, the profiles remain similar to themselves in time. This was done with times varying by a factor of approximately eight. It was established that, within the experimental accuracy, the shock behaves in a pseudo-stationary manner.

Because of the pseudo-stationary character of the phenomenon it is convenient to present the results in the non-dimensional $(x/\alpha, y/\alpha)$-plane. The shock profiles obtained experimentally are given in this manner in figure 3. In the interests of

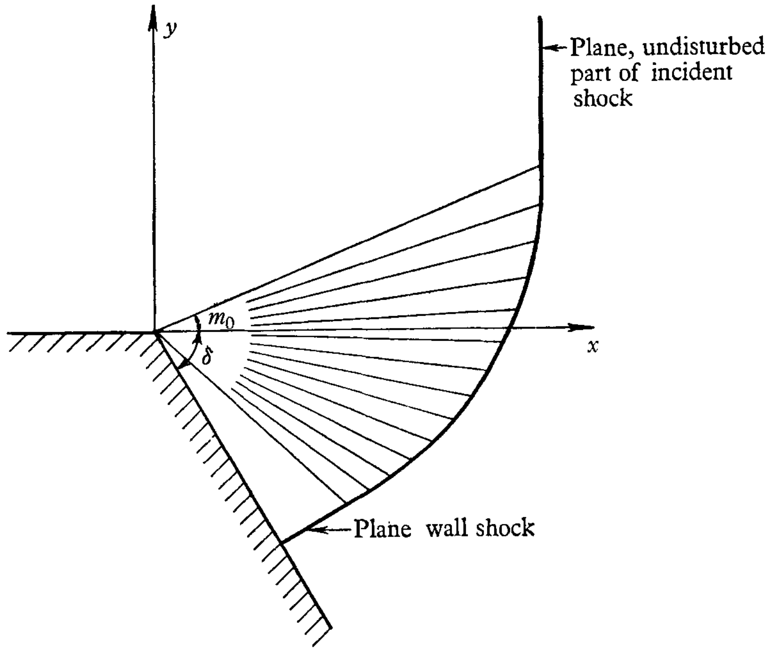

FIGURE 1. Diffraction on a convex corner.

clarity the results for $M_0 = 1{\cdot}2$, $1{\cdot}5$ and $2{\cdot}0$ are shown separate from, and to a larger scale than those for $M_0 = 3{\cdot}0$ and $4{\cdot}0$. The results for all the corners are superimposed, the thin radial lines indicating the wall positions for the different tests. The corner is at the origin and the circle of unit radius represents conditions for the wave to become sonic. The theoretically predicted shock curves (including the plane, wall shocks) are shown dotted in figure 3.

The first point of importance which is seen from the experimental results is that the shock profiles for the various corners do not form a single curve with only plane wall shocks perpendicular to the wall and tangent to the main shock curve. At any one Mach number the shocks for the various corners do, however, form an envelope, and this envelope will be considered as the main shock curve and is independent of corner angle. The envelope is more clearly defined at the higher Mach numbers. Those parts of the shock, for any given corner, which do not lie on this envelope will be referred to as wall shocks in the discussion that follows. It should be noted that the shock shapes for $M_0 = 2{\cdot}95$ given by Griffith & Brickl (1953) do not form as definite an envelope as was found in the present tests for $M_0 = 3{\cdot}0$.

A striking result to be seen from the experimental curves is that in no case does the diffracted shock become vanishingly weak even at an initial Mach number of only 1·20. This matter is taken up again in the next section.

The first marked discrepancy between the experimental and theoretical predictions is the point at which the shock curvature starts. This point is defined in terms of the angle m_0 in the theory (see figure 1), and in the experiments by the

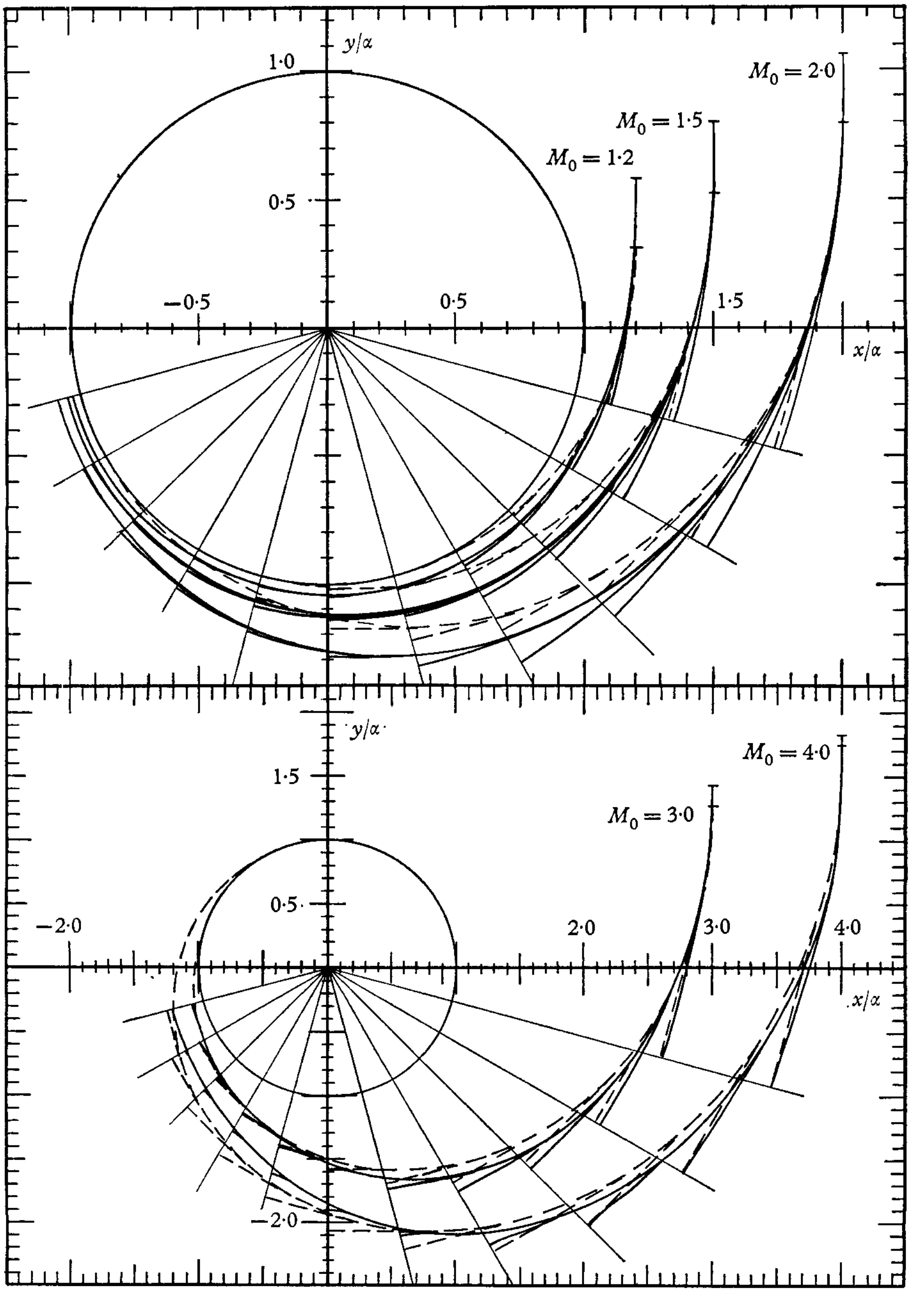

FIGURE 3. Shock wave profiles: ---, theory; —, experiment.

point where the reflected sound wave intersects with the incident shock (see figure 4*a*). These points are shown in figure 3 by short horizontal lines, the lower of the two being the theoretical prediction. Consider the situation shown in figure 4*a*. The incident wave moves with velocity u_0, the centre of the reflected sound wave with velocity u_1, the wave front itself moving with a velocity of a_1 relative to the gas. It may easily be shown from the geometry of the figure that

$$\tan m_0 = \frac{a_1}{u_0}\left\{1-\left(\frac{u_0-u_1}{a_1}\right)^2\right\}^{\frac{1}{2}}.$$

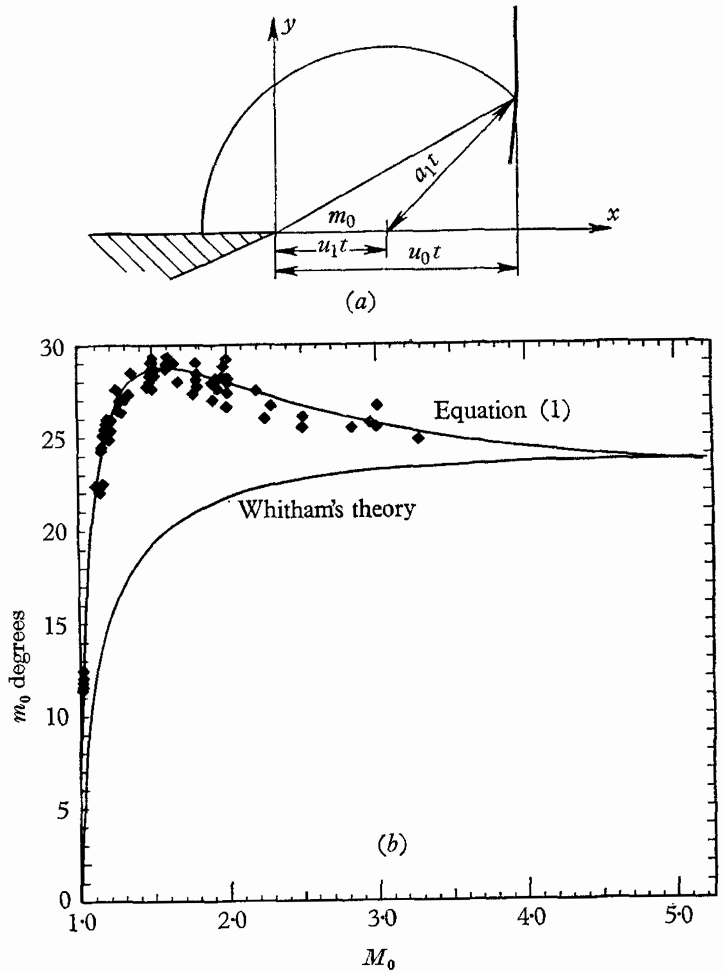

FIGURE 4. The starting-point of shock curvature.

Writing $u_0/a_0 = M_0$ and $u_1/a_1 = M_1$ and substituting from the normal shock relations gives

$$\tan^2 m_0 = (\gamma-1)(M_0^2-1)\{M_0^2+2/(\gamma-1)\}/(\gamma+1)M_0^4. \tag{1}$$

The variation of m_0 with M_0 as given by equation (1), the experimental results and Whitham's result are shown in figure 4*b*. Equation (1) satisfies the variation satisfactorily. The difference between the two theoretical curves was a reason (implied by Whitham) for expecting his theory to be inaccurate for $M_0 < 3{\cdot}0$. (At $M_0 = \infty$ Whitham's theory gives $m_0 = 23{\cdot}94°$ whereas (1) gives $22{\cdot}21°$.)

Consider now the comparison between the main shock curves, i.e. the envelopes of the shock profiles obtained for different corner angles (see figure 3). For $M_0 = 1{\cdot}2$ and $1{\cdot}5$ the theoretical and experimental curves move farther apart with increasing corner angle, the theoretical curve having the higher curvature. It therefore meets the limiting circle at a point corresponding to a smaller corner angle; this results because, in practice, the wave does not attenuate as rapidly. At $M_0 = 2{\cdot}0$ the same discrepancy is noted but there is an indication of the experimental curve approaching the theoretical curve at the higher corner angles. This tendency is even more marked for the $M_0 = 3{\cdot}0$ curve and, in fact, the two curves eventually cross for a corner angle of between 120 and 135°. At $M_0 = 4{\cdot}0$ this cross-over occurs at a much smaller angle (approximately 75°), the curves then diverge and subsequently reconverge to cross over once again at the largest angle. This second cross-over is to be expected since the shock Mach number is low there and it is known that at low Mach numbers the wave attenuates slower than predicted theoretically. The limiting condition will thus again be reached at much larger corner angles than those given by the theory.

The final consideration in this section is the behaviour of the wall shock. The theory predicts that the wall shock is plane, normal to the wall, and tangent to the main shock curve.

For all the Mach numbers employed the distance between the main shock curve and the point where the wall shock meets the wall is greatest for the smallest corner angle. This is qualitatively in agreement with the theoretically predicted behaviour. At the lower Mach numbers (1·2, 1·5 and 2·0) and the larger corner angles ($> 90°$) this deviation is not measurable as the wall shock and the main shock curve very nearly coincide, because the shock is approaching the sonic value.

At low Mach numbers (1·2 and 1·5) the wall shock is curved over its whole length; the distance from the wall to where it meets the main shock curve is much greater than in the theoretical case. This tendency is still visible for $M_0 = 2{\cdot}0$, particularly at the smaller angles, but is less marked.

For $M_0 = 3{\cdot}0$ the wall shock is, in general, nearly plane and the situation is qualitatively similar to that of the theory, the lengths of the wall shock also being roughly the same.

For corner angles greater than 30° and $M_0 = 4{\cdot}0$ the situation changes: the wall shock is shorter than predicted theoretically and, since it separates tangentially from the main shock curve, a point of inflexion occurs so that it can reach the wall at a right angle. For corner angles greater than 90° the point of inflexion occurs very close to the main shock curve and the wall shock is very small. This effect is clearly visible in figure 2*b*, plate 1. It is also clearly noticeable for tests at large corner angles and $M_0 = 3{\cdot}0$ and is probably present for the smaller corner angles but cannot be determined reliably.

From the preceding discussion it is clear that the model of a plane wall shock tangent to the main shock curve is, in general, a rough approximation. This arises since the theory cannot avoid concentrating the curvature over a relatively small portion of the shock. Lighthill (1949) has shown in his linearized theory that this concentration is valid, particularly for the stronger shocks. It is because of this that Whitham concluded that his theory should be more accurate for the stronger shocks. The better agreement found between his theory and experiment for $M_0 = 3{\cdot}0$ than for $M_0 = 1{\cdot}2$ may be ascribed to this effect. It is interesting to note that Lighthill's theory predicts a point of inflexion in the shock curve, near the wall, for $M_0 > 2{\cdot}531$ (see figure 1, Lighthill 1949). It has been noted above that such a point of inflexion is visible. The conclusions drawn by Lighthill regarding the shape of the shock are thus valid even when the corner angle is large. The interesting result is that the point of inflexion occurs for the large corner angles where the shock Mach number at the wall is relatively low.

3. Wall shock Mach number

The comparisons between theory and experiment are given in figure 5. The two-shock theory given by Parks (1952) is included for the 15 and 30° corners. It is inapplicable at the larger corner angles because of the separation of the flow at the corner.

For the 15° corner the two-shock theory gives values higher than Whitham's theory. The agreement between the experimental results and Whitham's theory is excellent over the whole Mach number range.

The same situation is apparent for the 30° corner, although at the lower end of the Mach number range the experiments give Mach numbers definitely higher than those of Whitham's theory. This shows the start of the tendency for the wall Mach number to decay slower than predicted theoretically in this range. The results of Parks's experiments are also shown in this figure. In general these give a lower wall Mach number than those obtained by this author.

The comparisons between theory and experiment are similar for the remaining corners: the experimental curve is a straight line with a slope less than that of the major portion of the theoretical curve, the actual wall Mach numbers being higher than the theory for the lower incident shock Mach numbers and vice versa. This behaviour is in accordance with the trends established by Lighthill. He showed that, for cases where there is no point of inflexion, maximum weakening will occur when the diffracted shock is a straight line normal to the wall. Therefore for the curved shocks which occur in the low Mach number range it is to be expected that the wall Mach numbers would be higher. Once a point of inflexion is allowed greater weakening is possible. This effect is clearly shown in figure 6. The points plotted are average values for a number of tests. For $M_0 = 1{\cdot}5$ and $2{\cdot}0$ where the wall shock is convex to the still air the wall shock Mach number decays slower than predicted by Whitham's theory. At $M_0 = 3{\cdot}0$ the wall shock is approximately straight, being close to Whitham's theoretical model and the agreement between theory and experiment is good. For the 15° corner and $M_0 = 4{\cdot}0$ both the main shock curve and the wall shock are different from those of Whitham's theory yet the overall effect is such as to make the theoretical and experimental wall Mach numbers agree. (This result may be deduced from figure 3.)

As the wall angle is increased to 75° so the actual wall Mach number becomes less than that given by the theory, in spite of the main shock curve causing the opposite tendency. This is due to the appearance of the point of inflexion and the greater weakening allowed thereby. Above 75° the shape of the main shock wave

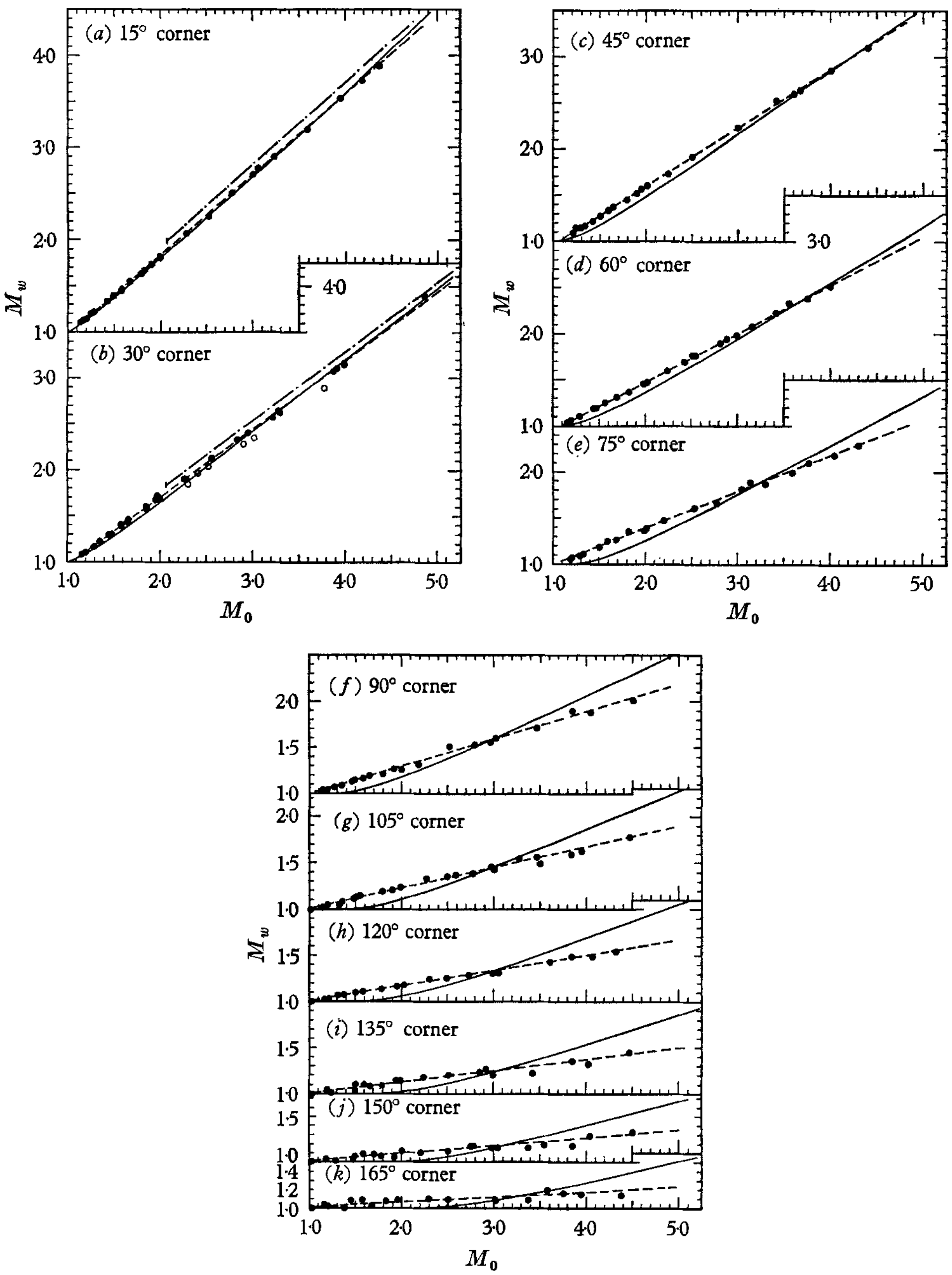

FIGURE 5. Variation of wall shock Mach number with incident shock Mach number. –·–·–, two-shock theory; —, Whitham's theory; – – –, experiment; ○, Parks's experimental results.

and the effect of the inflexion work together to give low wall shock Mach numbers. This results in the rapid divergence of the $M_0 = 4{\cdot}0$ curves in figure 6. As the shock again approaches the theoretical curve, at 165°, so this effect is lessened and the curves approach each other.

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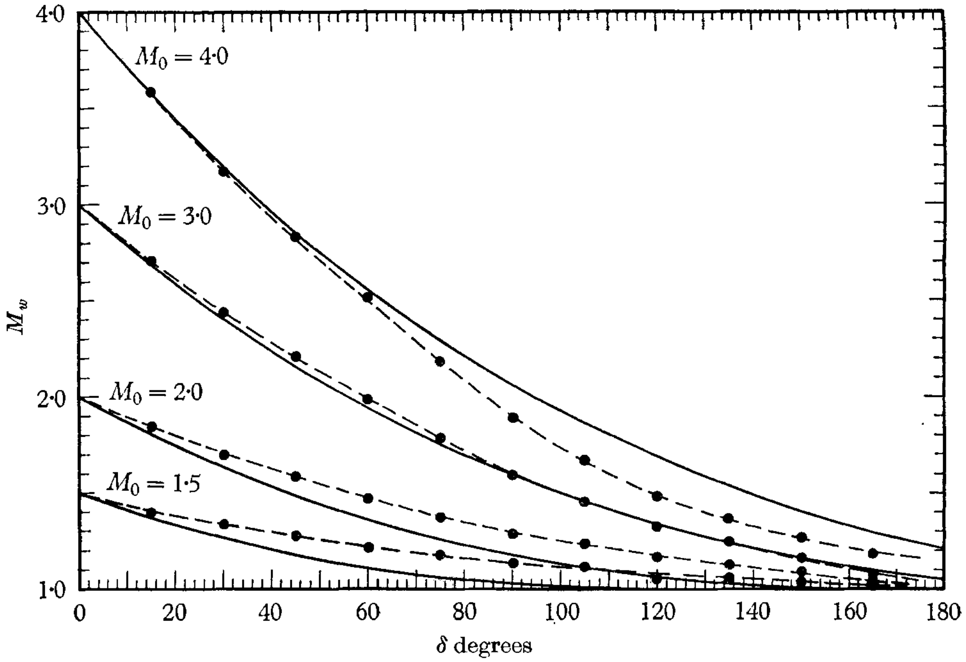

FIGURE 6. The decay of the wall shock Mach number with corner angle. —, Theory; – – –, experiment.

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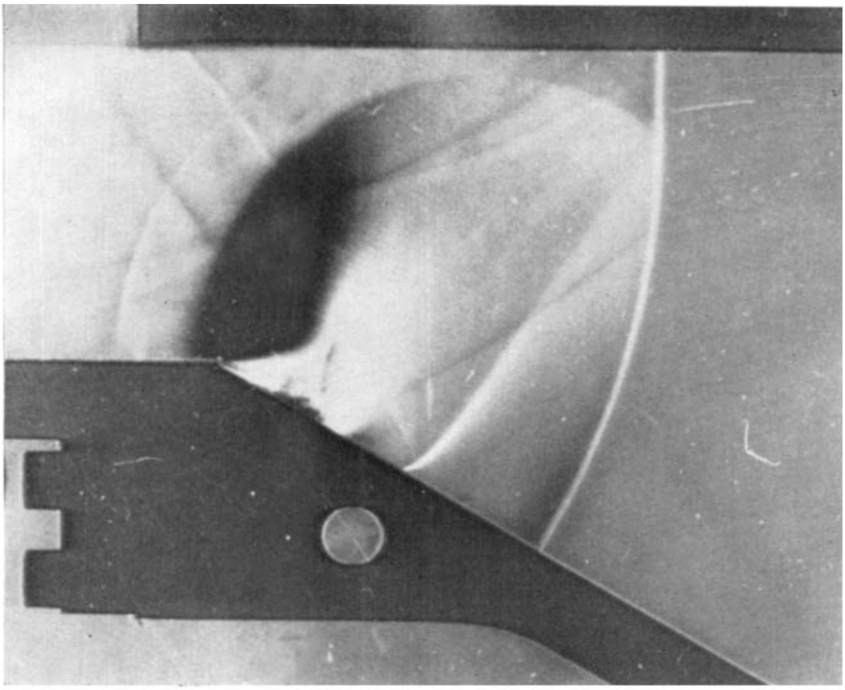
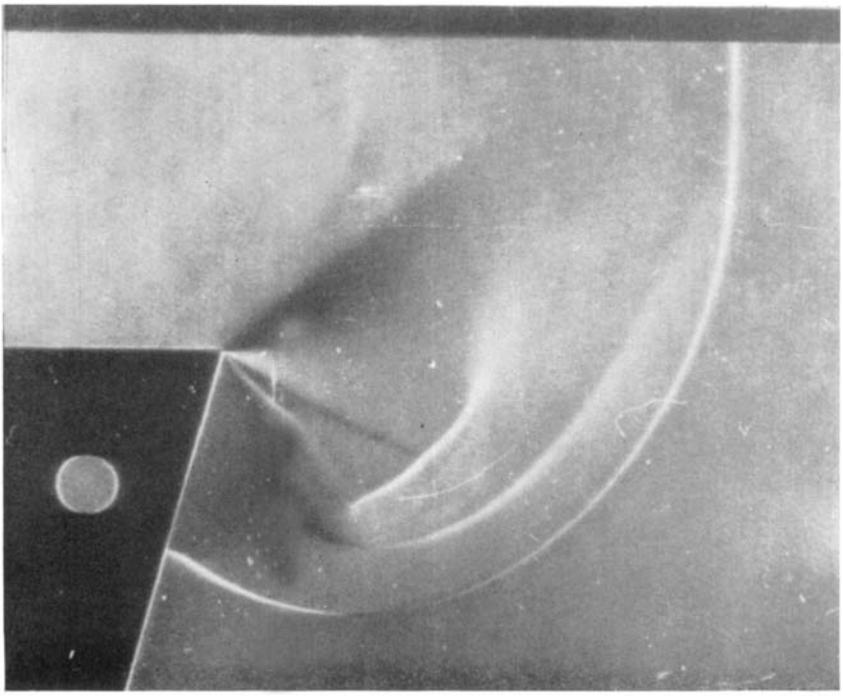


(*a*)



(*b*)

FIGURE 2. Schlieren photographs of shock diffraction on plane walled convex corners. (*a*) $M_0 = 2 \cdot 0$, (*b*) $M_0 = 4 \cdot 0$.